Acme Engineering College

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Set A (2075-04-09) Hints & Solution

Section I

1. (c)

$$KE = \frac{3}{2}KT$$

or, $K = \frac{2E}{3T} = ML^2T^{-2}\theta^{-1}$

2. (a)
3. (b)
4. (c)
5. (b)
6. (c)
7. (a)
8. (d)

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Where $\mu = A + \frac{B}{\lambda^2}$

Here, $\lambda_r > \lambda_v$ so $\mu_r < \mu_v$

$\therefore$ f will be maximum for red

9. (d)

$\frac{e/m}{2e/4m} = 2 : 1$

10. (c)
11. (d)

$$\tan\theta = \cot 2\theta = \tan\left(\frac{\pi}{2} - 2\theta\right)$$

or, $\theta = n\pi + \frac{\pi}{2} - 2\theta$

$\therefore$ $\theta = (2n + 1)\frac{\pi}{6}$

12. (b)

$$\tan^{-1}\alpha + \tan^{-1}\beta = \tan^{-1}\left(\frac{\alpha + \beta}{1 - \alpha\beta}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{1 - \frac{1}{6}}\right)$$

$$= \tan^{-1}1 = \frac{\pi}{4}$$

13. (a)

Comparing to $x^2 + y^2 + 2gx + 2fy + c = 0$

$g^2 = f^2 = c = a^2$

So, touches both axes

14. (c)

$(x - 4)^2 + y^2 < (x - 2)^2 + y^2$

$\Rightarrow -8x + 16 < -4x + 4$

$\Rightarrow 12 < 4x \Rightarrow 3 < x \Rightarrow Re(z) > 3$

15. (c)

Since parallel support means they have same or opposite direction.

16. (c)

Two planes taken together given a lines so we have three lines

x = 1, y = 2 …..(1)

x = 1, z = 3 …..(2) and y = 2, z = 3

All of them passes through (1, 2, 3)

17. (a)

$\begin{vmatrix} p & 2 \\ 1 & 1 \end{vmatrix} \neq 0 \Rightarrow p - 2 \neq 0$

$\therefore$ $p \neq 2$

18. (d)

$\lim_{x\to\infty} \frac{e^x - 1}{x} \left[\frac{\infty}{\infty}\right]$

$= \lim_{x\to\infty} e^x$

$= e^\infty = \infty$ (does not exist)

19. (b)

Total no. of attempts = 10 × 10 × 10 = 1000.

There is only one successful and hence no. of unsuccessful attempts = 1000 – 1 = 999

20. (b)

$$\int \frac{1}{\frac{e^x - e^{-x}}{2}}dx = 2\int \frac{e^x}{(e^x)^2 - 1}dx$$

$$= 2\int \frac{dt}{t^2 - 1} = 2.\frac{1}{2}\log\left(\frac{t-1}{t+1}\right) + c$$

$$= \log\left(\frac{e^x - 1}{e^x + 1}\right) + c$$

Put $e^x = t$

$e^x = t$

$\therefore$ $e^x\,dx = dt$

21. (c)

1 mole of $H_2O = 3N_A$ atoms

0.1 mole of $H_2O = 0.3N_A$ atoms

22. (c)

O.N. of metal in alloys is O

23. (d)

In CH_3Cl, carbon is surrounded by different atoms so the net dipole is not zero.

24. (b)

He has smallest atomic size so it has highest first ionization energy.

25. (b)

H_3PO_3 in a dibasic acid

26. (b)

HSO_4^- can both donate and accept proton so it is both acid and base

27. (c)
    Cl_2 with hot and conc. NaOH gives $NaClO_3$
28. (a)
    Hypo is $Na_2S_2O_3 . 5H_2O$
29. (d)
    Na_2SO_4 is not used to remove hardness of water.
30. (b)
    If two halogen atoms are bounded to same carbon atom then it is called gem – dihalide.
31. (a)
    The addition of HBr to alkene starts with addition of H^+ -ion so it is an electrophilic addition reaction.
32. (c)
    The product is benzene which on ozonolysis gives glyoxal

33. (b) 34. (b) 35. (c) 36. (d) 37. (c) 38. (b)
39. (c) 40. (b) 41. (b) 42. (b) 43. (b) 44. (b)
45. (a) 46. (b) 47. (a) 48. (b) 49. (c) 50. (b)
51. (b) 52. (a) 53. (c) 54. (d) 55. (b) 56. (c)
57. (b) 58. (b) 59. (c) 60. (b)

Section – II

61. (b)
    $$V_{av} = \frac{2V_1V_2}{V_1 + V_2} = \frac{2 \times 40 \times 60}{40 + 60} = 48 \text{ km/hr}$$
62. (b)
    $$\text{Net force experienced} = \frac{\text{Total impulse}}{\text{Total time}} = \frac{m\Delta v}{t} = 30\text{N}$$
63. (a)
    Loss in PE = Gain in KE
    $$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$
    or, $mgh = \frac{1}{2} \times \frac{2}{5} mr^2 \times \frac{v^2}{r^2} + \frac{1}{2} mv^2$
    or, $mgh = \frac{2mv^2 + 5mv^2}{10}$
    or, $v = \sqrt{\frac{10}{7} gh}$
64. (b)
    $$\frac{1}{2} mv^2 = \frac{1}{2} kx^2$$
    or, $x = v\sqrt{\frac{m}{k}} = 1.5\sqrt{\frac{0.5}{50}} = 0.15 \text{ m}$
65. (d)
    Thermal capacity = ms = 40 × 0.2 = 8 cal /°c
    $= 4.2 \times 8 \text{ J/}^o\text{c} = 33.6 \text{ J/}^o\text{c}$
66. (c)
    $\theta_n = \frac{\theta_c + \theta_i}{2}$ $\therefore \theta_c = 2\theta_n - \theta_I = -20^o c$
67. (c)
    $$\phi = \frac{2\pi x}{\lambda}$$
    or, $\lambda = \frac{2\pi x}{\phi} = \frac{2\pi \times 0.4}{1.6\pi} = 0.5\text{m}$
    $\therefore \lambda = \frac{v}{\lambda} = 660 \text{ Hz}$
68. (a)
    $$f' = \frac{9}{8} f$$
    App. Frequency $(f') = f \times \frac{(v + u)}{(v - u)}$
    or, $\frac{9}{8} f = f \times \frac{(v + u)}{(v - u)}$
    or, $9v - 9u = 8v + 8u$
    or, $v = 17u$
    or, $u = \frac{v}{17} = \frac{340}{17} = 20 \text{ m/s}$
69. (c)
    Work done = change in energy
    $$= \frac{1}{2}\left(C + \frac{C}{2}\right) V^2 = \frac{3}{4} CV^2$$
70. (d)
    $E = V + Ir$
    $12 = V + 60 \times 5 \times 10^{-2}$
    $\Rightarrow V = 9 \text{ volt}$
71. (c)
    Torque $(\tau) = MB \sin\theta = \vec{M} \times \vec{B}$
72. (b)
    $$\frac{f_a}{f_l} = \frac{\left(\frac{\mu_g}{\mu_l} - 1\right)}{(\mu_g - 1)}$$
    or, $\frac{2}{f_l} = \frac{\frac{1.5}{1.25} - 1}{1.5 - 1}$
    or, $f_l = 5 \text{ cm}$
73. (b)
    n = 8, D = 0.72 cm R = 300
    $r = \frac{D}{2} = 0.36 \text{ cm}$
    $\lambda = ?$
    We have (for transmitted system)

$\lambda = \frac{2r^2}{(2n-1)R} = \frac{2\times(0.36)^2}{(2\times8-1)\times300}$

$= 5760\times10^{-8}$ cm

$\therefore \quad \lambda = 5760\times10^{-10}$ m

74. (a)

$KE = hf - \phi = \frac{hc}{\lambda} - \phi$

$= \frac{6.62\times10^{-34}\times3\times10^{8}}{5000\times10^{-10}\times1.6\times10^{-19}} - 1.9$

$= 2.48 - 1.9 = 0.58$ ev

75. (d)

No. of half lives, $n = \frac{t}{T} = \frac{6400}{800} = 8$

$\therefore \quad \frac{N}{N_o} = \left(\frac{1}{2}\right)^8 = \frac{1}{256}$

76. (b)

$a\cos^2\frac{B}{2} + b\cos^2\frac{A}{2} = a\left(\frac{1+\cos B}{2}\right) + b\left(\frac{1+\cos A}{2}\right)$

$= \frac{a + a\cos B + b\cos A + b}{2}$

$= \frac{a+b+c}{2}$

77. (b)

Put $x = \tan\theta$

$\sin\left\{\tan^{-1}\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) + \cot^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right\}$

$= \sin\left\{\tan^{-1}\tan\left(\frac{\pi}{2} - 2\theta\right) + \cos^{-1}\cos2\theta\right\}$

$= \sin\left(\frac{\pi}{2} - 2\theta + 2\theta\right) = 1$

78. (b)

$\left(\frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \ldots\right) - \left(\frac{y^2}{1!} + \frac{y^4}{2!} + \frac{y^6}{5!} + \ldots\right)$

$= (e^{x^2} - 1) - (e^{y^2} - 1)$

$= e^{x^2} - e^{y^2}$

79. (d)

Normal is $x + 2y = k$, which passes through centre (1, 0) of circle. So, $k = 1$

$\therefore \quad x + 2y = 1$

80. (d)

For $(x-2)^2 = y - 1$

Length of LR = 1

So, (d) is incorrect

81. (a)

$$\begin{vmatrix} 1 & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & 1 & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & 1 \end{vmatrix}$$

$$= \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 0$

82. (d)

Planes,

$4x - 4y + 2z + 2 = 0$ and $4x - 4y + 2z + 3 = 0$

Distance $= \frac{3-2}{\sqrt{16+16+4}} = \frac{1}{6}$

83. (b)

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$f(t) = t^2 - 2$

$\therefore \quad f(x) = x^2 - 2$

84. (a)

$\vec{BC} = \vec{i} + \vec{j}, \quad \vec{AB} = \vec{i} - \vec{j}$

$\vec{AC} = \vec{AB} + \vec{BC} = \vec{i} - \vec{j} + \vec{i} + \vec{j} = 2\vec{i}$

85. (c)

$f'(x) = \frac{1}{2} - \frac{2}{x^2} \qquad \therefore f''(x) = \frac{4}{x^3}$

$f'(x) = 0 \Rightarrow x^2 = 4 \qquad \therefore x = -2, 2$

For $x = 2$, $f''(x) > 0$

So, $x = 2$ point of local minima

86. (a)

$y = \log_{\sqrt{e}}(\sin x) = \frac{1}{1/2}\log_e \sin x = 2\log_e \sin x$

$\therefore \quad \frac{dy}{dx} = 2.\frac{1}{\sin x}\cos x = 2\cot x$

87. (d)

$\int \frac{x\,dx}{\sqrt{4-x^4}}dx = \frac{1}{2}\int\frac{1\,d(x^2)}{\sqrt{4-(x^2)^2}}$

$= \frac{1}{2}\sin^{-1}\left(\frac{x^2}{2}\right) + c$

88. (d)

Solving, $x^2 = 2 - x^2$

or, $x^2 = 1$

$\therefore$ $x = \pm 1$

$$A = \int_{-1}^{1} (2 - x^2 - x^2)\, dx$$

$$= \left[2x - \frac{2x^3}{3}\right]_{-1}^{1} = \frac{8}{3}$$

89. (d)

Sum = $1 + 2.2x + 3.(2x)^2 + 4.(2x)^3 + \ldots$

$$= \frac{1}{1 - 2x} + \frac{1.2x}{(1 - 2x)^2} = \frac{1}{(1 - 2x)^2}$$

90. (c)

$3(x - 1)^2 + 4(y + 1)^2 = 5 + 3 + 4 = 12$

$\therefore$ $\frac{(x - 1)^2}{4} + \frac{(y + 1)^2}{3} = 1$

$\therefore$ Centre (T),

$c = \sqrt{1 - \frac{3}{4}}$ (T)

foci $(1 \pm \sqrt{4 - 3}, -1) = (1 \pm 1, -1)$

$\Rightarrow$ $(2, -1)$ and $(0, -1)$

So, (c) is not correct.

91. (c)

$S_2 = \frac{V_1 \times S_1}{V_2} = \frac{50 \times 24}{100} = 12\ N = 6\ M$

92. (d)

Cu^{++} + $2e \longrightarrow Cu$

1mole 2 mole = 2×96500 c

0.3 mole $0.3 \times 2 \times 96500$ c

$= 6 \times 96500$ c

93. (c)

pOH = $-\log[OH^-] = -\log 0.015 = 1.82$

pH = $14 - 1.82 = 12.18$

94. (b)

$NH_3 + 3Cl_2 \longrightarrow NCl_3 + 3HCl$

95. (c) 96. (c) 97. (b) 98. (c) 99. (d) 100. (d)